Circuit Double Cover Conjecture - 3-edge-coloring, weight decomposition, Kotzig frame
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![Figure: A cubic graph with a 3-edge-coloring](image-url)
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**Figure:** A cubic graph with a 3-edge-coloring
CDC induced by 3-edge-coloring

Figure: Double covering by three bi-colored even subgraphs

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**DEFINITION**

Let \( w : E(G) \mapsto \{1, 2\} \) such that the total weight of every edge-cut is even (eulerian weight).
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A family \( \mathcal{F} \) of circuits of \( G \) is a faithful cover with respect to \( w \) if each edge \( e \) is contained in precisely \( w(e) \) members of \( \mathcal{F} \).
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\begin{array}{c c c}
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![Diagram showing faithful cover concept]
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**Question**
Does every bridgeless cubic graph have a faithful cover with respect to every eulerian weight?

**Answer:** No!
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Black edges: weight 2
Green edges: weight 1
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- $G$ is planar (Seymour);
- $G$ is 3-edge-colorable (Seymour)
- $G$ is Petersen-minor free (Alspach, Goddyn, Z.).
A Key Lemma: 3-edge-coloring and faithful cover

(Seymour) If a cubic graph $G$ is 3-edge-colorable, then $G$ has a faithful circuit cover with respect to every $(1, 2)$-eulerian weight.
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**Proof.** Let $w : E(G) \mapsto \{1, 2\}$ be an eulerian weight of a cubic graph $G$ and $c : E(G) \mapsto \{\text{Red, Black, Green}\}$ be a 3-edge-coloring of $G$. Here $E_w=1$ (weight one edges) induces an even subgraph, $C_{\text{Red} - \text{Black}}$ be Red-Black bi-colored even subgraph, $C_{\text{Red} - \text{Green}}$ be Red-Green bi-colored even subgraph, $C_{\text{Green} - \text{Black}}$ be Green-Black bi-colored even subgraph, Then a faithful even-subgraph cover:

$$\{ E_w=1 \cup C_{\text{Red} - \text{Green}}, E_w=1 \cup C_{\text{Red} - \text{Green}}, E_w=1 \cup C_{\text{Green} - \text{Black}} \}$$

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3-edge-coloring and faithful cover

Figure: A cubic graph with a 3-edge-coloring.
3-edge-coloring and faithful cover

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3-edge-coloring and faithful cover

\[ \Delta = \sum_{w \in E} \Delta_{Red} - \Delta_{Green} + \sum_{w \in E} \Delta_{Green} - \Delta_{Black} \]
3-edge-coloring and faithful cover

\[ \begin{align*}
&= \left( \begin{array}{ccc}
1 & & 1 \\
& + & \\
& & 
\end{array} \right) \\
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\[
\begin{aligned}
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\end{aligned}
\]
3-edge-coloring and faithful cover

\[ \begin{align*}
E_w &= 1 \\
\Delta C_{\text{Red} - \text{Green}} &+ \Delta C_{\text{Red} - \text{Green}} + \Delta C_{\text{Green} - \text{Black}}
\end{align*} \]
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\[
\begin{align*}
1 & = 1 + 1 + 1 \\
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**An example.**

![Graph Diagram]
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**An example.**

![Diagram](image_url)
Weight decomposition: two copies of 3-edge-colorable graphs

\[ S = G_1 + G_2 \]

\[ G_1 = G[S \cup P] \quad G_2 = G[S \cup R] \]

Figure: Both \( G_1 \) and \( G_2 \) are 3-edge-colorable
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\( \mathcal{F}_1 \): faithful cover of \( G_1 \) (cover \( S \) once),
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\( F_1 \cup F_2 \) is a CDC of \( G \).
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Definition. The oddness of a cubic graph $G$ is the smallest integer $t$ such that every 2-factor of $G$ has at least $t$ odd components.
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Oddness $= 0 \implies$ 3-edge-colorable $\implies$ CDC.

Oddness $= 2$ (including Petersen graph) $\implies$ CDC. (Häggkvist and McGuinness 2005, Huck 2004 (CAP))

Oddness $= 4$ $\implies$ CDC. (Ye, Z.)

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If \((G, 2)\) has a weight decomposition:
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Weight decomposition into three 3-colorable graphs

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Which graph has such decomposition?
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Examples: $K_4$, $K_3$, $3$, Heawood graph, and dodecahedron, (and many others).

**Figure:** $K_4$: a Kotzig graph: bi-colored edges induce a Hamilton circuit.
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Definition A cubic graph $H$ is called a Kotzig graph if $G$ has a circuit double cover consisting of three Hamilton circuits. Examples: $K_4$, $K_{3,3}$, Heawood graph, and dodecahedron, (and many others).

Figure: $K_4$: a Kotzig graph: bi-colored edges induce a Hamilton circuit
Figure: Heawood graph
Dodecahedron Graph

Figure: Dodecahedron
A spanning subgraph $H$ of $G$ is a **Kotzig frame** of $G$.
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Kotzig Frame

A spanning subgraph \( H \) of \( G \) is a **Kotzig frame** of \( G \) if \( H \) is a subdivision of a Kotzig graph.

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A spanning subgraph $H$ of $G$ is a **Kotzig frame** of $G$ if $H$ is a subdivision of a Kotzig graph.

(Goddyn, Häggkvist and Markström) If a graph $G$ has a Kotzig frame, then $G$ has a CDC.
Figure: $K_4$ as a Kotzig Frame: three copies of Hamiltonian graphs

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Circuit Double Cover Conjecture - 3-edge-coloring, weight decomposition
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Circuit Double Cover Conjecture - 3-edge-coloring, weight decomposition
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**Conjecture** (Häggkvist and Markström) If $G$ contains a spanning subgraph $H$ such that $G/E(H)$ is even and every component of $H$ is an even circuit or a subdivision of Kotzig graph, then $G$ has a CDC.
**Conjecture** (Häggkvist and Markström) If $G$ contains a spanning subgraph $H$ such that $G/E(H)$ is even and every component of $H$ is an even circuit or a subdivision of Kotzig graph, then $G$ has a CDC.

Several cases of the above conjecture have been verified by Goddyn, Häggkvist and Markström, Ye and Z., Z. and X. Zhang in various papers.